A STUDY CONCERNING THE FLOW OF A HIGH-TEMPERATURE GAS AROUND BODIES WITH AXIAL SYMMETRY AND SOME DISCONTINUITY AREAS ON THE SURFACE

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UDC 533.6.011

An analysis is made of the error in calculating the density of a high-temperature gas without taking into account the effect of internal density jumps and the inaccuracy with which the location of interference fringes has been measured. The resultant density distribution is shown across one section of a cone in an air stream behind a shock wave.

For studies of a high-temperature gas stream around various models, of the boundary layer, of the heat transfer, etc. one often uses optical methods of measurement. In these models the determination of local characteristics of axially symmetrical inhomogeneities reduces to the solution of the Abel integral equation. The usual procedure for obtaining a solution is to approximate the measured function within a given zone by a known function: a constant, a straight line, a second-order or higher-order curve. Because of the inherent inaccuracy of such an approximation, an error is then introduced in the determination of local characteristics. Experience has shown that this error is small in the case of continuous functions but becomes significant near locations where the inhomogeneity parameters change suddenly and the approximating curve deviates from the actual curve most. The problem of reducing the error in the calculation of density jumps by applicable methods arises in the evaluation of test data on the density of a high-temperature gas around a model body in a shock tube.

The measurements were made with an interferometer and a diffraction grating in a model IAB-451 instrument [1]. Typical interferograms are shown in Fig. 1a, b. The pattern produced in a stream around a sphere by interference fringes of diffraction orders +1 and −1 are shown in Fig. 1a. To the

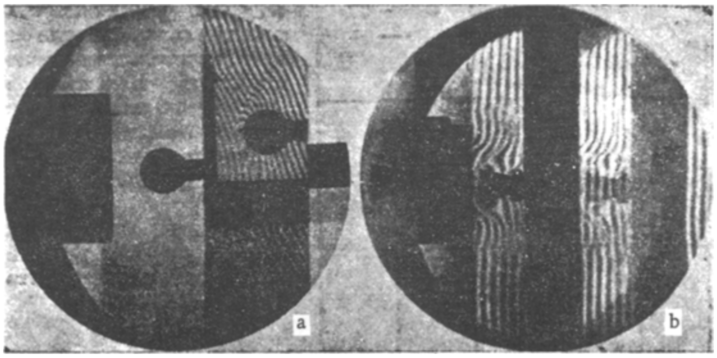

Fig. 1. (a) Interference pattern in a stream around a sphere in a shock tube (interference fringes of orders +1 and −1), (b) interference pattern in a stream around a conic segment in a shock tube (interference fringes of orders 0 and +1).

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 3, pp. 414-418, March, 1973. Original article submitted April 28, 1971.

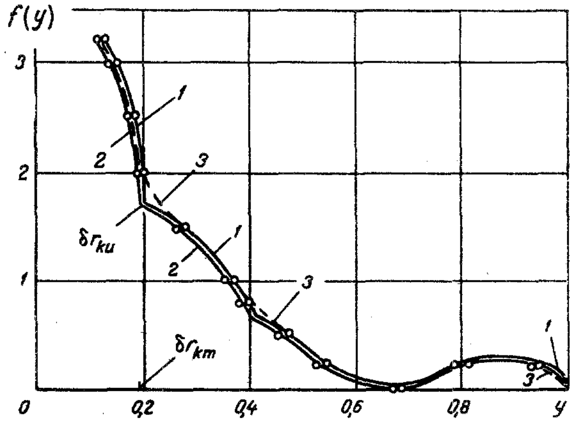

Fig. 2. Distribution of interference fringe shifts across an inhomogeneity section in an air stream around a cone behind a shock wave: maximum shifts of interference fringes (1), minimum shifts of interference fringes (2), curve of fringe shifts without accounting for internal density jumps (3). Points represent test values of interference fringe shifts.

left of the principal image there appears an image similar to one obtained by the light-spot method. It is produced by interference between fringes of orders 0 and +1.

The pattern obtained by interference fringes of orders 0 and +1 in a stream around a conic segment is shown in Fig. 1b.

In this case the stream structure was quite complex. The leading shock wave travelling through low-density air ($\rho \approx 2 \cdot 10^{-5}$ g/cm^3) at a velocity of ~3000 m/sec constituted the inhomogeneity boundary. Behind this leading wave followed a rarefaction wave constituting a zone of large negative density gradients. The test model was located within the Mach cone, i.e., the zone of constant stream parameters. The density jump at the model was that internal jump behind which the density was to be determined. If the density under the bottom of the model was to be determined, then attention had to be paid to the zone of large gradients appearing at the separation and the transition to a bottom vacuum.

The shift of interference fringes has been plotted in Fig. 2 for a section through an axially symmetrical inhomogeneity emerging around the cone. The points represent test values of the shift obtained through a $\lambda/4$, based on measurement of the location of bright fringes, dark fringes, and the boundaries between them. The solid lines represent the maximum fringe shift (1) and the minimum fringe shift (2). The difference between them $2\delta f(y)$ results from the inaccuracy involved in measuring the shift of interference fringes. The dashed line is a result of quadratic interpolation of test values without acconnting for internal density jumps.

In the course of this study the authors compared the results of calculations by the methods in [2] and in [3]. The values of $[\rho(r_i)-\rho_0]$ were calculated from curves 1-3 by various methods disregarding the effect of a jump. The final density graph is shown in Fig. 3. The spread of values is rather wide. The method of linear interpolation for the zone of the rarefaction wave behind the leading shock wave ($r_i \le 0.8$) has yielded negative values for the gas density in the stream. The values obtained by different methods for the density within the zone of the strong leading shock wave differed by 10-20%.

It is to be noted that quadratic interpolation of the test function without accounting for the radical relation f(y) (curve 3 in Fig. 2) within zones near jumps of the refractive index has yielded a large error in the density calculation: the density discontinuity has been transformed into a high-gradient zone (curve 3 in Fig. 3).

After completion of calculations based on the shadow pattern and the refraction of interference fringes, the location of density jumps was determined and appropriate corrective terms were added according to the method in [4]. The jump coordinate is $r_k = 0.2$ on curve 1 and $r_k = 0.196$ on curve 2. The difference between those two values represents the error δr_k in the determination of the jump coordinate. It is to be noted that δr_{ku}, which results from an intersection of curves between the jump and the left-hand branches of the fringe shift curves 1 and 2, is equal to δr_{km} incurred directly during measurement of the jump location on the shadow pattern. The addition of corrective terms has, in effect, eliminated the discrepancy between both results. Values obtained by the different methods agree after correction. Meanwhile, the corrected values for curves 1 and 2 in Fig. 2 differ appreciably, especially for the zones of high density gradients. This has made it possible to estimate the error in the density determination due to the inaccuracy with which the shift of interference fringes and the location of a density jump are measured. Evidently, if corrective terms are added in the calculation of axially symmetrical inhomogeneities with density jumps, then the total error is entirely determined by the measurement error. If no corrective terms are added, however, then the error due to internal jumps becomes predominant in calculations for the zones near density jumps.

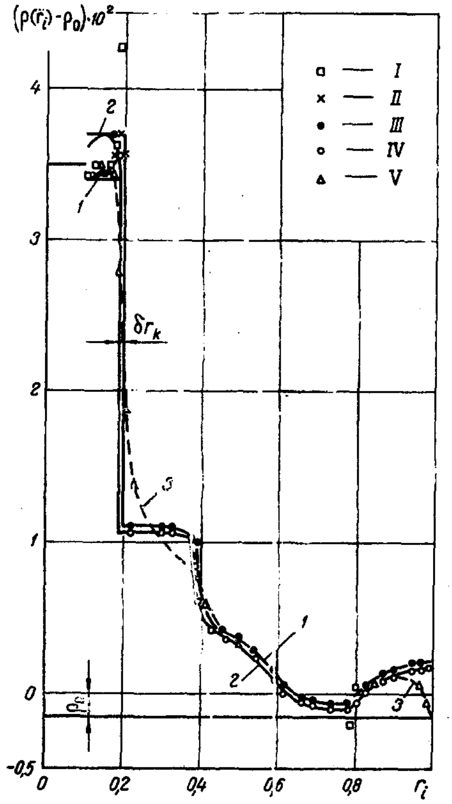

Fig. 3. Calculated density distributions ($kg \cdot sec^2/m^4$) across an inhomogeneity section, in an air stream around a cone behind a shock wave: by the method in [2] with N = 25 (I), by the method in [3] with N = 50 (II), by the method in [3] with N = 25 for curve 1 (III), by the method in [3] with N = 25 for curve 2 (IV), for curve 3 in Fig. 2 (V). Density curves calculated from curves 1, 2, 3 in Fig. 2 after an addition of corrective terms (1, 2, 3 respectively). Values of ρ_0 are indicated between vertical arrows.

This has made it possible to define the ranges of inhomogeneity which require corrective terms for density calculations. In practice these terms are needed in calculations for one or two zones near a jump, while these ranges are much smaller in methods based on quadratic approximation than in methods based on linear approximation.

An analysis of many calculations similar to those described here and made for various test objects has shown that the error due to an internal jump may vary widely in magnitude. In the case of linear approximation this error is large and, therefore, the addition of corrective terms if most often justified. In the case of quadratic approximation one may often omit corrective terms, especially when the measurement error is sufficiently large while the jump is small and zones far from it are of main interest.

In practical measurements it is worthwhile to estimate the magnitudes of corrective terms and of measurement errors so as to be able, when considering the specific purpose of an experiment, to decide whether the former are necessary or the latter should be accounted for. When precision measurements are required, there arises the problem of improving the accuracy in determining the optical path differences within various regions of an inhomogeneity and in determining the locations of density jumps. For studying inhomogeneities with large internal density gradients, it is necessary to use a method by which values of the measured function can be found at more points.

NOTATION

ρ	is the density in a given inhomogeneity;
ρ_0	is the density in the unperturbed stream;
r_i	is the coordinates of a point where the density is calculated, referred to the radius of the inhomogeneity;
y	is the referred coordinates of the point of entrance of the light beam into the inhomogeneity;
f(y)	is the function of the shift of interference fringes;
δf(y)	is the error in the value for f(y) due to measurement inaccuracy.

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